UNIWERSYTET
IM. ADAMA MICKIEWICZA
W POZNANIU

Section of Logic and Cognitive Science

Institute of Psychology
Adam Mickiewicz University
in Poznań

Mariusz Urbański

Tableaux, Abduction and Truthlikeness

RESEARCH REPORT

Szamarzewskiego 89, 60-589 Poznań, Poland
tel.: (48 61) 829 23 07, (48 61) 829 21 07, fax: (48 61) 829 21 07

Tableaux, Abduction and Truthlikeness.
Research Report

Mariusz Urbański*
Section of Logic and Cognitive Science
Institute of Psychology
Adam Mickiewicz University
Szamarzewskiego 89
60-589 Poznań, Poland
e-mail: Mariusz.Urbanski@amu.edu.pl

0 The structure

The present report consists of three sections. In Section 1 a short characteristics of STM is given together with an informal discussion of the most important notions introduced here. In Section 2 it is shown how generation of abducibles works with STM (within the framework of purely computational account on abduction, however). In Section 3 there are outlined some connections between STM and Theo A.F. Kuipers account on scientific theories and his notion of truthlikeness. It is shown also how theory-expansion and theory-revision, which are crucial for this analysis, can be performed within the framework of STM.

1 Synthetic Tableaux

Synthetic Tableaux Method (STM) is a certain decision method (see Urbański (2001a) and (2002a) for details). The fundamental ideas underlying STM can be traced back to the L. Kalmár's proof of the completeness of CPC. Recall, that in this proof one uses the fact that every valid formula is entailed by every consistent set made up of all of its propositional variables or their negations. However, the original proof of Kalmár is system-dependent: it can be applied to every logic which validates certain theorems. STM generalizes its idea. Intuitively it can be said that a proof of a formula *A* consists of all the possible attempts of "synthesizing" *A* or *non-A* on the basis of the consistent sets of their subformulae (with the sets of basic constituents of *A* or their negations interpreted as representing "initial conditions" or "basic assumptions"). A formula in question is proved if and only if all such attempts end with *A*. *Mutatis mutandis*, the same pertains to STM as a model-seeking procedure for sets of formulae. Thus, "one way or another" is the shortest description of the ideas underlying STM.

In the present paper we use STM for the first time also as a model-seeking[1] method which enables deciding problems concerning satisfiability (of sets of formulae) and entailment as well as validity of formulae.

The notion of well-formed formula (wff for short) is defined as usual as well as the notion of a subformula of a given wff. We use Sub(*A*) to represent the set of all the subformulae of a formula *A* and Sub(*X*) to represent the union of sets of subformulae of all the elements of a set *X* of wffs. By a *literal* we mean a propositional variable or the negation of a propositional

* The author was supported by Foundation for Polish Science.
[1] Here and later on we will be using the term 'model' in a bit loose way, in the sense that a set of formulae has a model iff it is satisfiable.

variable. If φ is a propositional variable, the literals φ and '$\neg\varphi$' will be called *associates* and *based on* the variable φ.

1.1 STM – general

Synthetic tableaux are defined as families of sequences of formulae (so called 'synthetic inferences'). We introduce these notions first giving their exact definitions and then clearing them up with examples and some discussion.

DEFINITION 1

A finite sequence $\mathbf{s} = s_1, \ldots, s_n$ of formulae is a *synthetic inference of a set* $X = \{A_1, \ldots, A_k\}$ *of wffs* iff the following conditions hold:

(1) **subformula condition**
for any formula s_i of $\mathbf{s}$, s_i is an element of Sub(X) or the negation of an element of Sub(X);

(2) **starting condition**
s_1 is a literal;

(3) for any formula s_g of $\mathbf{s}$, s_g satisfies exactly one of the following:

(a) **introduction of literals**
s_g is a literal and the associate to it does not appear in $\mathbf{s}$;

(b) **introduction of compound formulae**
s_g is derivable from a certain set of formulae such that each element of this set occurs in $\mathbf{s}$ before s_g;

(4) one of the following holds:

(a) **successful closure**
all of the formulae $A_1, \ldots, A_k$ are terms of $\mathbf{s}$, or

(b) **unsuccessful closure**
for at least one A_i (i=1, …, k): $\neg A_i$ is a term of $\mathbf{s}$.

A synthetic inference of a set X is *successful* iff it meets successful closure condition (4a). A synthetic inference of a set X is *unsuccessful* iff it meets unssucceful closure condition (4b).

Inference rules are the following:

$\neg A \,/\, A\rightarrow B$	$B \,/\, A\rightarrow B$	$A, \neg B \,/\neg(A\rightarrow B)$
$A \,/\, A\vee B$	$B \,/\, A\vee B$	$\neg A, \neg B \,/\neg(A\vee B)$
$\neg A \,/\, \neg(A\wedge B)$	$\neg B \,/\, \neg(A\wedge B)$	$A, B \,/\, A\wedge B$
$A \,/\, \neg\neg A$		

Example 1

An unsuccessful synthetic inference $\mathbf{s}$ of a set $X=\{p\rightarrow(q\wedge r), (p\rightarrow q)\vee(p\rightarrow r)\}$:

$\mathbf{s} = p, \neg q, \neg(q\wedge r), \neg(p\rightarrow(q\wedge r)), r, p\rightarrow r, (p\rightarrow q)\vee(p\rightarrow r)$

A synthetic inference **s** of a set X of wffs can be interpreted as an attempt to find a model of X. In the case of CPC what is relevant to this end are literals of **s** (as they determine the valuations under which the elements of X are true or false). All other terms of **s** are introduced as derivable from what has been previously established in **s** - thus, in the end, from literals (in the above example all the non-literal formulae are entailed by the set $\{p, \neg q, r\}$). The subformula condition warrants that no 'irrelevant' formula appears in **s**. Closure conditions allow to determine wheter an inference in question forms a model of X or not. Notice, that in view of the assumed consequence relation, introduction of literals condition guarantees that the set of formulae of **s** is consistent.

A synthetic inference **s** of a set X is *completed* iff for every A_i in X: either A_i is a term of **s** or $\neg A_i$ is a term of **s** (thus successful synthetic inferences are always completed).

The reason for introducing this last notion is the following. It is of course enough to have a negation of an element of X at a certain inference **s** to conclude that **s** does not form a model of X. Nevertheless, one may be reasonably interested in how many and which elements of X are 'vicious', and this will become important in subsequent sections.

Our main notion is given by the following definition:

DEFINITION 2

A family Ω of finite sequences of formulae is a *synthetic tableau for a set* $X=\{A_1, \ldots, A_k\}$ *of wffs* iff each element of Ω is a synthetic inference of a set X (they are called *paths* of Ω) and the following hold:

(1) **uniform start**
there exists a propositional variable φ such that the first term of every sequence in Ω is a literal based on φ;

(2) for every sequence **s** in Ω the following holds:
if s_i is a literal, then:

(a) **fairness of branching**
Ω contains a certain synthetic inference **s'** such that **s** and **s'** do not differ up to their i-1th terms and s_i' is an associate to s_i;

(b) **binary branching**
if $i>1$, then for each synthetic inference **s'** such that **s** and **s'** do not differ up to their i-1th terms: either $s_i'=s_i$ or s_i' is an associate to s_i.

The clause 2 of the above definition can be in more exact terms expressed as follows:

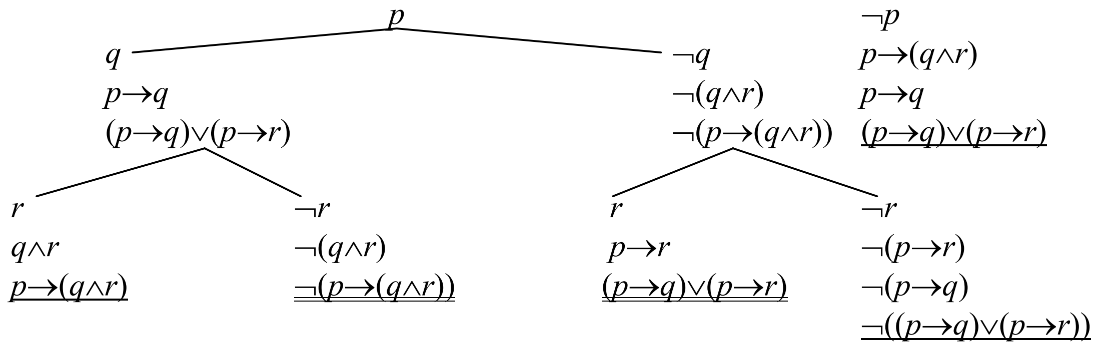
(2') for every sequence $\mathbf{s}=s_1,\ldots,s_n$ in Ω the following holds:
if s_i is a literal, then:

(a') Ω contains a certain synthetic inference $\mathbf{s'}= s_1',\ldots,s_m'$ such that s_i' is an associate to s_i and, if $i>1$, then $s_h'=s_h$ for $h=1,\ldots,i-1$;

(b') if $i>1$, then for each such a synthetic inference $\mathbf{s'}= s_1',\ldots,s_r'$ in Ω that $s_h'=s_h$ for $h=1,\ldots,i-1$, the following holds: $s_i'=s_i$ or s_i' is an associate to s_i.

Although synthetic tableaux are defined as sets of sequences of wffs we are going to represent them as a tree-like structures, branching downwards, as in the example given below.

Example 2
A synthetic tableau for a a set $X=\{p\to(q\wedge r), (p\to q)\vee(p\to r)\}$:



The last formula of a synthetic inference is indicated by underlining – single in the case of successful closures, double in the case of unsuccessful ones.

A synthetic tableau for a set X of wffs is a family of synthetic inferences of X that are interconnected by uniform start, fairness of branching and binary branching conditions. Uniform start together with binary branching guarantee that a tableau branches on literals only. Recall, that literals are the only formulae on paths of a tableau that are introduced without 'inferential justification' (*i.e.*, they are not entailed by any other formula). They may be interpreted as assumed premises of a reasoning represented by a certain path. Fairness of branching guarantees that introduction of literals is 'fair' with respect to the underlying semantics. Due to this condition introduction of a literal at a path forces branching of this path with simultaneous introduction of an associate of the literal.

These conditions, together with subformula condition of definition 1, may be interpreted as forming a very restricted version of cut rule. It is restricted to propositional variables that are subformulae of the elements of X and could be called 'atomic' or 'literal' cut.

With such a tree-like representation of tableaux their synthetic character becomes easily visible. Interpreting literals as the simplest pieces ('bricks') of information, every path of a tableau for X is an attempt to synthetize, as goals, the most compound pieces (the elements of X or their negations) on the basis of those 'bricks'. Fairness of branching is to the effect that all the possible relevant combinations of 'bricks' are taken into account, thus in a tableau search for a model of X becomes systematic[2]. By 'relevant' we mean here 'sufficient to derive goals': in the above example '$\neg p$' is enough to derive all the elements of X, so there is no need to introduce any other literal at the path beginning with '$\neg p$'.

1.2 Soundness and completeness

Soundness and completness of STM with respect to the underlying semantics are given by the following theorem:

[2] Interpretation of STM as a systematic search procedure can be easily expressed in terms of Inferential Erotetic Logic (see Urbański (2001b) for details).

THEOREM 1 (completness-soundness)
A set X of wffs is satisfiable iff there exists a synthetic tableau Ω for X such that at least one path of Ω is successful.

Moreover, it can be proved that there exists a synthetic tableau Ω for X such that at least one path of Ω is successful iff every tableau for X has this feature.

As a special case of theorem 1 one gets:

THEOREM 2
$A_1, \ldots, A_k \models B$ iff there exists a synthetic tableau Ω for a set $X = \{A_1, \ldots, A_k, B\}$ such that for every element **s** of Ω at least one of the following holds:

(1) for at least one A_i (i=1, …, k): $\neg A_i$ is a term of **s**;
(2) B is a term of **s**.

We will be speaking of a tableau for a set $X = \{A_1, \ldots, A_k, B\}$ as of *a synthetic tableaux for a derivation of a formula B on the basis of wffs* $A_1, \ldots, A_k$ (and, respectively, of synthetic inferences of such derivations). Clauses 1 and 2 of the above theorem express conditions of successful closure of synthetic inferences of B on the basis of $A_1, \ldots, A_k$. Let us state them explicitly together with unsuccessful closure condition:

DEFINITION 3
A synthetic inference of a derivation of a formula B on the basis of wffs $A_1, \ldots, A_k$ is:

(1) *successful* iff one of the following holds:
  (a) for at least one A_i (i=1, …, k): $\neg A_i$ is a term of **s**;
  (b) B is a term of **s**;
(2) *unsuccessful* iff all of the formulae $A_1, \ldots, A_k, \neg B$ are terms of **s**.

Proofs of theorems 1 and 2 are easy modifications of the proof of the following theorem (cf. Urbański (2002a)), which is also a special case of theorem 1:

THEOREM 3
$\models B$ iff there exists a synthetic tableau Ω for a singleton set $X=\{B\}$ such that every path of Ω is a successful.

1.3 Partial tableaux, entanglement

In what follows we will make use of the notion of a *partial* synthetic tableau for a given set of formulae:

DEFINITION 4
A family Ω of finite sequences of formulae is a *partial synthetic tableau for a set X of wffs* iff each element of Ω is a synthetic inference of a set X (they are called *paths* of Ω) and uniform start and binary branching conditions (of definition 2) are met.

Example 3
A partial synthetic tableau for a a set $X=\{p\rightarrow(q\wedge r), (p\rightarrow q)\vee(p\rightarrow r)\}$:

	p	
q		$\neg q$
$p\to q$		$\neg(q\wedge r)$
$(p\to q)\vee(p\to r)$		$\neg(p\to(q\wedge r))$
$\neg r$		r
$\neg(q\wedge r)$		$p\to r$
$\underline{\underline{\neg(p\to(q\wedge r))}}$		$\underline{(p\to q)\vee(p\to r)}$

In the example 3 it is shown a partial synthetic tableau for a a set $X=\{p\to(q\wedge r), (p\to q)\vee(p\to r)\}$ such that it consists of unsuccessful paths of the tableau of example 2.

A partial tableau for a given set X may be interpreted as representing a search for a model of X with some fixed (or assumed as known) information (this is the effect obtained by dropping uniform start and fairness of branching conditions). In the above example what is fixed is the starting literal p (as a result, uniform start condition is not fulfilled) and '$\neg r$' in the presence of q as well as r in the presence of '$\neg q$' (these violate fairness of branching condition).

One important notion we will be using in the next section is the notion of an entanglement of a formula in a certain synthetic inference:

DEFINITION 5

Let $\mathbf{s} = s_1,\ldots, s_n$ be a synthetic inference of set $X = \{A_1, \ldots, A_k\}$ of wffs. A formula C is *entangled in* **s** iff the following holds:

(1) there exists $i = 1, \ldots, n$ such that $C = s_i$;

(2) (a) there exists a propositional variable φ such that $C = \varphi$ or $C = \neg\varphi$ and there exists $g = 1, \ldots, k$ such that $s_i = A_g$, or

(b) there exists $j>i$ such that s_j is derivable from a certain set Y of formulae such that each element of this set occurs in **s** before s_j, and $s_i\in Y$.

Thus a formula C is entangled in **s** iff C is a term of **s** and either C is an element of X (if C is a literal), or C is used as a premise of an inference rule to obtain another term of **s**. Informally speaking, formulae entangled in an inference **s** are precisely these which are relevant for the success (or the lack of it) of **s**, that is, the formulae from which the elements of X (or their negations) can be derived.

In the example 1 all the formulae of the inference **s** are entangled except '$\neg(p\to(q\wedge r))$' and '$(p\to q)\vee(p\to r)$'. In the next example the situation is more complicated:

Example 4

A synthetic inference **s** of a set $X=\{(p\to q)\vee(p\to r)\}$:

$\mathbf{s} = \neg q, \neg p, p\to r, r, p\to q, (p\to q)\vee(p\to r)$

Here possible sets of entangled formulae are $\{\neg p, p\to q\}$ and $\{\neg p, p\to r\}$ (note, that r cannot be considered as entangled formula as it does not precede $p\to r$ in **s**). As the inference rules defining the underlying notion of derivability are non-deterministic, the entanglement is non-deterministic as well.

2 Abduction

2.1 Peirce

Accordind to C.S. Peirce the general structure of an abductive reasoning is the following (cf. Magnani (2001)):

> The surprising fact, *C*, is observed.
> But if *A* were true, *C* would be a matter of course.
> ―――――――――――――――――――――――
> Hence, there is reason to suspect that *A* is true.

In his early, 'syllogistic' theory, deduction, abduction and induction are distinct forms of reasoning each of which corresponds to a certain form of a syllogism. Peirce's own examples are the following (cf. Aliseda (1997)):

Deduction

Rule	All the beans from this bag are white.
Case	These beans are from this bag.
Result	These beans are white.

Here, truth of the Result is warranted by truth of the Rule and the Case.

Induction

Case	These beans are from this bag.
Result	These beans are white.
Rule	All the beans from this bag are white.

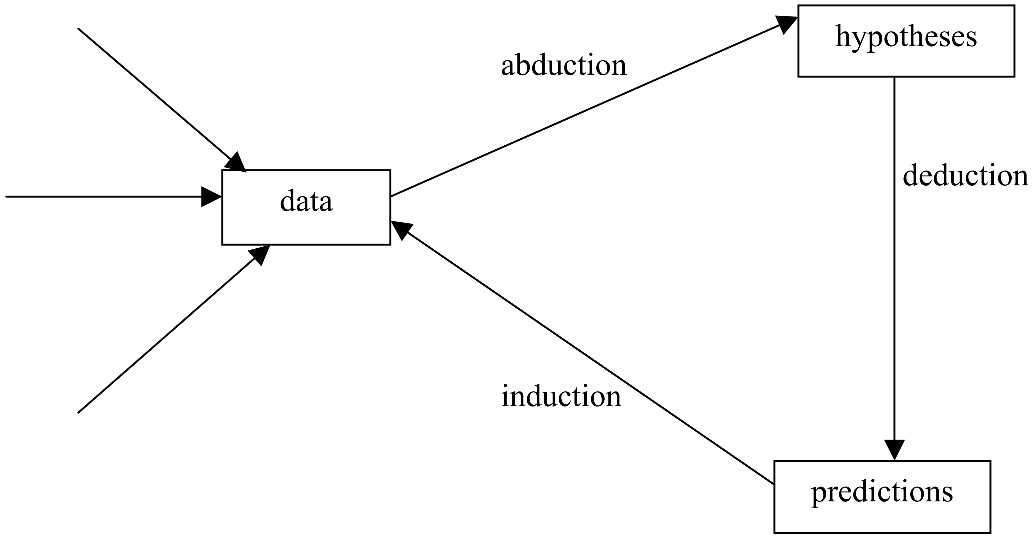
Rule produced as a conclusion of inductive reasoning is validated only in a 'long run' (and still it may be false).

Abduction

Rule	All the beans from this bag are white.
Result	These beans are white.
Case	These beans are from this bag.

In abductive reasoning, the Case is suggested, which may explain what is the connection between the Rule and the Result.

Later on Peirce proposed an 'inferential' theory. He considered abduction, deduction and induction as stages composing a method of logical inquiry, of which abduction is the beginning: *From its* [abductive] *suggestion deduction can draw a prediction which can be tested by induction* (Peirce, 5.171).



Induction, abduction and deduction as three stages of logical inquiry

New data, coming from some external source of information or obtained by inductive confirmation of earlier predictions, can give rise to new hypotheses. A circular structure of this model reveals non-monotonic character of this method of inquiry: it is possible that hypotheses already corroborated can be withdrawn in view of new data.

2.2 Abducibles

A very basic intuitions underlying the notion of abductive explanation, or an abducible, are expressed by the following definition:

DEFINITION 6
A formula H is an *abductive explanation* (an *abducible*) for a formula A with respect to the set of formulae X iff:

(C1) X *non* $\models A$, and
(C2) $X \cup \{H\} \models A$

The conditions C1 and C2 are the core ones. We can formulate a number of additional conditions which can be used to define different *styles* of abduction. Some examples include:

consistent	(C3)	$X \cup \{H\}$ is consistent
explanatory	(C4)	H *non* $\models A$
minimal	(C5)	H is weakest such explanation
preferential	(C6)	H is the best explanation according to some given preferential ordering

We are interested here in the core criteria. Let us formulate them in terms of STM:

DEFINITION 7
A formula H is an *abducible for a formula B with respect to the set of formulae $X = \{A_1, \ldots, A_k\}$* iff

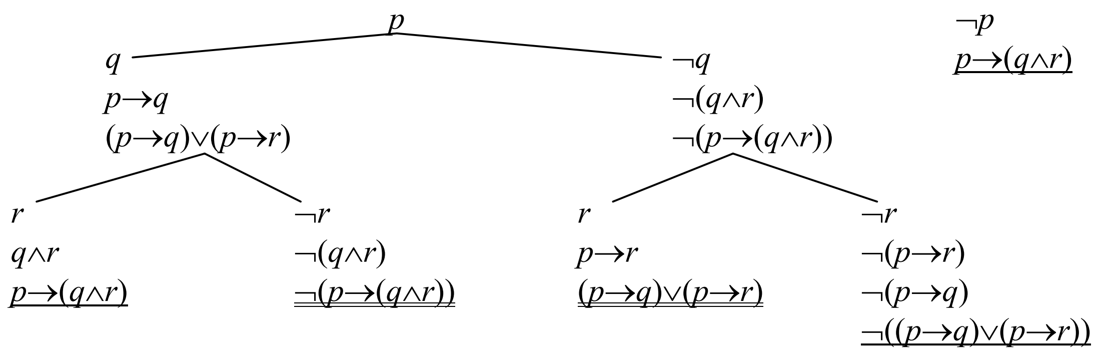
(C1’) there exists a synthetic tableau Ω for a derivation of a formula B on the basis of wffs $A_1, \ldots, A_k$ such that at least one path of Ω is unsuccessful and

(C2’) there exists a synthetic tableau Ω’ for a derivation of a formula B on the basis of wffs $A_1, \ldots, A_k$, H such that every path of Ω’ is successful.

2.3 Generation of abducibles

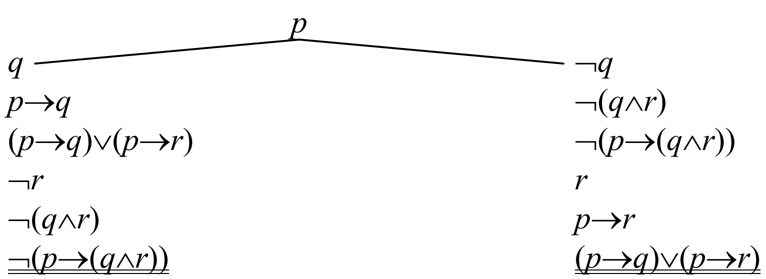
Consider the following tableau for a derivation of ‘$p\to(q\wedge r)$’ on the basis of ‘$(p\to q)\vee(p\to r)$’:

Example 3:



This tableau shows that it is not the case that ‘$(p\to q)\vee(p\to r)$’ entails ‘$(p\to(q\wedge r)$’ and, moreover, it provides relevant countermodels (so to speak, in a bit ‘loose’ way). The problem of how to fill the deductive gap can be approached *via* STM in a few ways. One of them is the following.

Take into considerations all the unsuccessful paths, that is, the following partial tableau for a derivation of ‘$(p\to(q\wedge r)$’ on the basis of ‘$(p\to q)\vee(p\to r)$’:



As all the non-literal formulae of a certain path are entailed by the set of its literals it is obvious that ‘responsible’ for the lack of success here are two combinations of literals: p, q, $\neg r$ and p, $\neg q$, r. Notice, that in both cases all the literals are entangled in the relevant inferences.

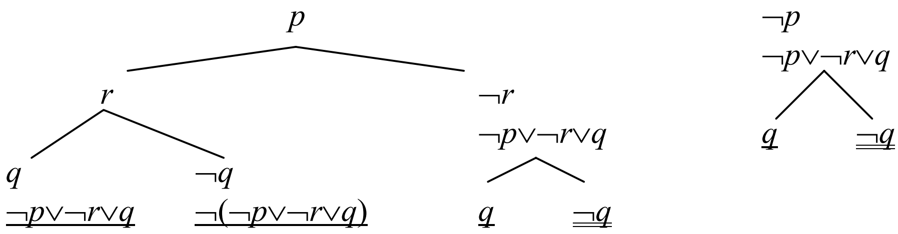
Now, for each unsuccessful path, build a conjunction of its literals and negate it (in the above example obtaining, e.g., $\neg(p\wedge q\wedge\neg r)$ and $\neg(p\wedge\neg q\wedge r)$). Build a conjunction of resulting formulae and compute its disjunctive normal form (DNF).

A DNF of ‘$\neg(p\wedge q\wedge\neg r) \wedge \neg(p\wedge\neg q\wedge r)$’ is
‘$(\neg p\wedge\neg p)\vee(\neg q\wedge\neg p)\vee(r\wedge\neg p)\vee(\neg p\wedge q)\vee(\neg q\wedge q)\vee(r\wedge q)\vee(\neg p\wedge\neg r)\vee(\neg q\wedge\neg r)\vee(r\wedge\neg r)$’.

It is easy to check that every disjunct (as well as any formula logically eqiuvalent to it) together with ‘$(p\rightarrow q)\vee(p\rightarrow r)$’ entails ‘$(p\rightarrow(q\wedge r)$’ that is, fills the deductive gap. As for the method – simply for any disjunct, take it as an additional premise and try to derive it at every unsuccessful path. As an example take ‘$r\wedge q$’ and try to extend the above tableau to the tableau for a derivation of ‘$(p\rightarrow(q\wedge r)$’ on the basis of ‘$(p\rightarrow q)\vee(p\rightarrow r)$’ and ‘$r\wedge q$’. Let us consider every path, starting from the left. At the first path there is already present a formula ‘$q\wedge r$’, so this path remains successful (clause 4a). The second path (an unsuccessful one) contains a formula ‘$\neg(q\wedge r)$’ and thus becomes successful by the clause 4b. The same holds for the third path. Moreover, with ‘$\neg(q\wedge r)$’ present at both third and fourth paths, the new tableau becomes shorter than the initial one.

Consider another example (taken from Aliseda(1997)), a tableau for derivation of q on the basis of $\neg p\vee\neg r\vee q$:

Example 4:



Here, the literals of unsuccessful paths are p, $\neg r$, $\neg q$ and $\neg p$, $\neg q$. Notice, however, that in the first case p is not entangled. Thus the relevant ‘conjunction of negated conjunctions of literals’ is $\neg(\neg r\wedge\neg q)\wedge\neg(\neg p\wedge\neg q)$. A DNF of this formula is: $(p\wedge r)\vee(p\wedge q)\vee(q\wedge r)\vee(q\wedge q)$. Again, every disjunct is an abducible.

We can define the following procedure for generation of abducibles:

> Let Ω be a synthetic tableau for a derivation of a formula B on the basis of wffs A_1, …, A_k such that at least one path of Ω is unsuccessful. Let $\mathbf{s}^1$, …,$\mathbf{s}^r$ ($r\geq 1$) be all the unsuccessful paths of Ω.
>
> 1. For $\mathbf{s}^1$:
>   a. determine the entangled literals of $\mathbf{s}^1$; let them be φ_1, …,φ_p ($p\geq 1$);
>   b. build a formula C_1 = ‘$\neg(\varphi_1\wedge \ldots\wedge\varphi_p)$’;

2. Perform step 1 for $\mathbf{s}^2$, …,$\mathbf{s}^r$ (if any);
3. Build a formula D = ‘$C_1 \wedge \ldots \wedge C_r$’;
4. Compute a disjunctive normal form of D;

As a result of step 4 one gets a formula of the form ‘$E_1 \vee \ldots \vee E_t$’, where every E_i (i=1, …,t) is a conjunction of literals.

Every such E_i (as well as every conjunction and disjunction of E_i’s) and every formula logically equivalent to it is an abducible for B with respect to the formulae $A_1, \ldots, A_k$.

The procedure is based on core abductive criteria (C1 and C2) and it may produce abducibles which are trivial or too strong, or inconsistent with the underlying theory. In order to make additional abductive criteria (as, e.g. C3 – C6) working we need some means for theory modification – expansion or revision.

3 Theory expansion, theory revision

In the last part of the paper we show how to interpret in terms of STM a conceptual apparatus[3] proposed by T.A.F. Kuipers in his *From Instrumentalism to Constructive Realism* (Kuipers (2000)) and how theory expansion and theory revision work within the STM-framework.

3.1 Propositional Descriptions, Theories and Truthlikeness

Let $EP=\{\varphi_1, \ldots, \varphi_n\}$ be a set of propositional variables, indicating ‘elementary possibilities’.
A *propositional description generated by EP* is a pair γ=<Γ, Γ=**t**>, where:

- Γ a set made up of literals based on the variables of *EP* (Γ is called a *constituent* of γ);
- Γ=**t** is a *truth-claim* of γ (the claim that all the elements of Γ are true).

In terms of STM: if **s** is a synthetic inference of a given set of formulae, Γ is a set of all the literals of **s** and the truth-claim of γ can be interpreted as a valuation that satisfies all the elements of **s** (this comes from the soundness-completeness theorem of STM).

Let *Mp* be a set of all the constituents of propositional descriptions generated by *EP*. Let $T \subseteq Mp$ be a set of all nomic possibilities, defined with respect to a previously chosen domain of natural phenomena (that is, the nomic truth).

- A (general) *theory X* is a pair <x, x=T>, where $x \subseteq Mp$ and x=T is the truth-claim of the theory X.
- A (general) *hypotesis Z* is a pair <z, $T \subseteq z$>, where $z \subseteq Mp$ and $T \subseteq z$ is the truth-claim of the hypothesis Z.

In terms of STM: if Ω is a canonical[4] synthetic tableau for a given set of formulae, *Mp* is the set of all the sets of literals of the elements of Ω. *T*, in turn, can be interpreted as the set of all the sets of literals of some partial tableau Ω’ (information that is ‘fixed’ here corresponds to the choice of domain of phenomena, which itself determines what is nomically possible). Thus we may say, although in a bit loose way, that theory differs from hypothesis in that

[3] We use it here with slight modifications.
[4] Cf. Urbański (2001b) or (2002a).

theory considers all the nomic possibilities while in case of hypothesis it is possible that some of them are beyond its scope.

Theory Y is at least similar (close) to the truth as theory X iff:
(1) y-T is a subset of x-T,
(2) T-y is a subset of T-x,
where x, y are subsets of *Mp*'s of theories *X*, *Y* respectively.

An interpretation of this notion in terms of STM is obvious.

In order to show how this apparatus works, Kuipers introduces a very simple example of an electrical circuit (see Kuipers (2000), p. 140) which can be easily analyzed in terms of STM.

What is more interesting from our point of view, however, is the problem of truth approximation, that is, of 'upgrading' theories in order to make them closer to the truth. Theory expansion and theory revision are crucial tools for that kind of upgrading.

3.2 Expansion

To expand a theory $X = \{A_1, \ldots, A_k\}$ with a formula B:

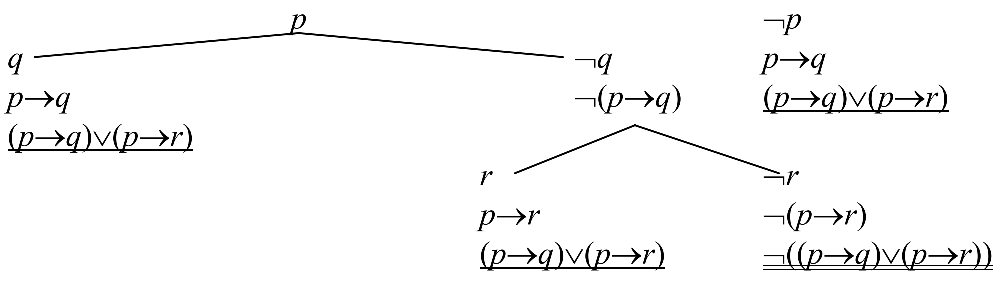
1. Build a synthetic tableau Ω for X;
2. Take a partial tableau Ω' of Ω that consists of these paths of Ω which are successful (that is, which have all the elements of X as their terms[5]);
3. For every path **s'** of Ω' compute B or $\neg B$ on **s'** with the condition that every term of **s'** must be an element of $\mathrm{Sub}(A_1)\cup\ldots\cup\mathrm{Sub}(A_k)\cup\mathrm{Sub}(B)$ or the negation of an element of $\mathrm{Sub}(A_1)\cup\ldots\cup\mathrm{Sub}(A_k)\cup\mathrm{Sub}(B)$.

If at least one path **s''** of the resulting (partial) synthetic tableau Ω'' for $X\cup\{B\}$ is such that all the elements of X as well as B are terms of **s''**, then the expansion is *consistent*. Otherwise it is inconsistent.

Example 5:

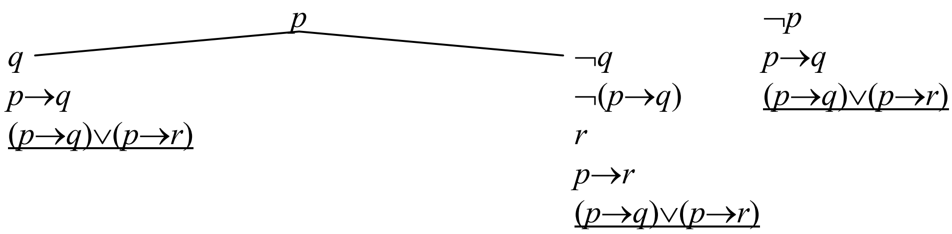
Consistent expansion of the theory $X=\{(p\rightarrow q)\vee(p\rightarrow r)\}$ with $r\wedge q$ (which is an abducible for $p\rightarrow(q\wedge r)$ with respect to $(p\rightarrow q)\vee(p\rightarrow r)$):

1. A tableau Ω for $\{(p\rightarrow q)\vee(p\rightarrow r)\}$

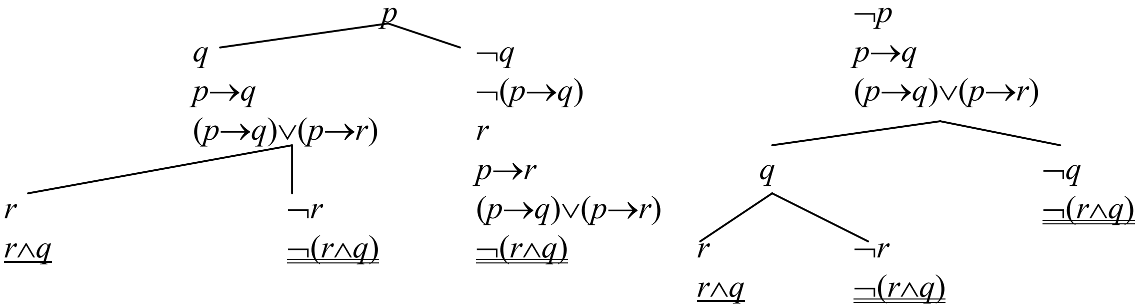


[5] Note, that if there is no such path this means that X itself is inconsistent.

2. A (partial) tableau Ω’, that consists of successful paths of Ω



3. A (partial) tableau Ω’’ for $\{(p\to q)\vee(p\to r)\} \cup \{r\wedge q\}$; Ω’’ results from Ω’ by computing either $r\wedge q$ or $\neg(r\wedge q)$ on each path of Ω’

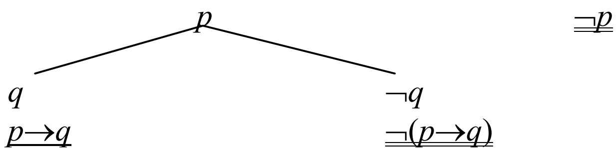


As there are two successful paths (that is, paths on which both $(p\to q)\vee(p\to r)$ and $r\wedge q$ appear), the expansion performed is consistent.

Example 6:

Inconsistent expansion of the theory $Y=\{p\to q, p\}$ with $\neg q$

1.



2.

p
q
$\underline{p\to q}$

3.

p
q
$p\to q$
$\underline{\underline{\neg\neg q}}$

As no path of the resulting partial tableau is successful, expanded theory $Y' = \{p \rightarrow q, p, \neg q\}$ is inconsistent.

3.3 Revision

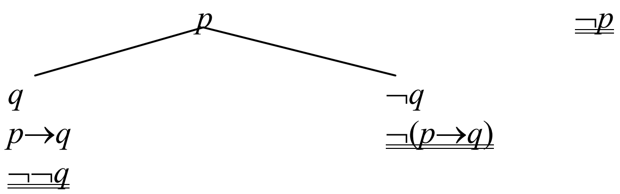
This is a two-step procedure. It consists of:

1. expansion of a given theory X with some formula (-ae), and
2. contraction of a resulting theory X', to eliminate, e.g., inconsistencies.

Example 7:

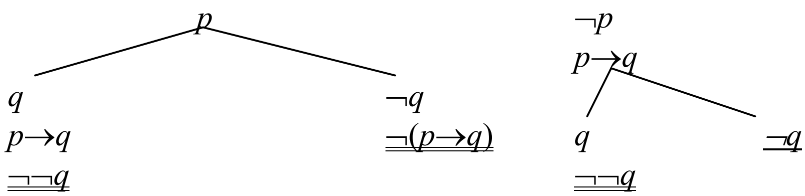
Revision of the theory $Y=\{p \rightarrow q, p\}$, to incorporate $\neg q$

1. Extension of $Y=\{p \rightarrow q, p\}$ to $Y'=\{p \rightarrow q, p, \neg q\}$

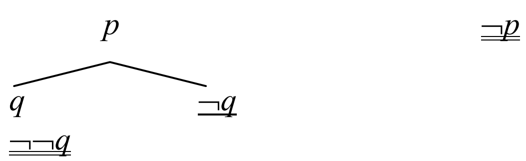


2. Contraction of the (inconsistent) theory $Y'=\{p \rightarrow q, p, \neg q\}$
As we prefer to have $\neg q$ in the revised theory there are two possibilities:

a. to contract Y' to $Z=\{p \rightarrow q, \neg q\}$



b. to contract Y' to $Z'=\{p, \neg q\}$



Both Z, Z' are consistent.

References:

Aliseda-Llera, A. (1997), *Seeking Explanations: Abduction in Logic, Philosophy of Science and Artificial Intelligence*, Institute for Logic, Language and Computation, University of Amsterdam.

Kuipers, T.A.F. (2000), *From Instrumentalism to Constructive Realism. On some relations between confirmation, empirical progress, and truth approximation*, Synthese Library, Vol. 287, Kluwer Academic Publishers, Dordrecht.

Magnani, L. (2001), *Abduction, Reason and Science. Processes of Discovery and Explanation*, Kluwer Academic Press/Plenum Publishers, Nowy Jork.

Urbański, M. (2001a), Remarks on Synthetic Tableaux for Classical Propositional Calculus, *Bulletin of the Section of Logic* 30, No. 4, pp. 194-204.

Urbański, M. (2001b), Synthetic Tableaux and Erotetic Search Scenarios: Extension and Extraction, *Logique et Analyse*, 173-174-175, pp. 69-91.

Urbański, M. (2002a), *Tabele syntetyczne a logika pytań* [*Synthetic Tableaux and the Logic of Questions*], Maria Curie-Skłodowska University Press, Lublin.

Urbański, M. (2002b), Synthetic Tableaux for Łukasiewicz's Calculus Ł3, *Logique et Analyse* 177-178, pp. 155-173.