Logic & Cognition
Questions: erotetic logic

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(Mostly adapted from Wiśniewski [2010, 2013])

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Do we reason with questions?

Q Who stole the tarts?

X It is one of the courtiers of the Queen of Hearts attending the afternoon tea-party who stole the tarts.

Q_1 Which of the Queen of Hearts' courtiers attended the afternoon tea-party?

Q Did John break into an ATM?

X_1 John broke into an ATM if and only if he went shopping.

X_2 John went shopping if and only if he's been seen in the mall

Q_1 Was John in the mall?

Do we really reason with questions? (Wiśniewski [1995])

Q Which town was the first capital of Poland?

X The town which was the first capital of Poland is now a town which is located about 50 km from Poznań.

Q_1 Which towns are located about 50 km from Poznań?

X_1 If the investigation concerning the murder of JFK was honest and many-sided, the real culprit has been found.

X_2 The real culprit has not been found.

Q Wasn't the investigation concerning the murder of JFK honest, or wasn't it many-sided?

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What is a question?

Bromberger [1992]

We ask questions for all sorts of reasons and with many different purposes in mind – e.g., to test someone's knowledge, to offer someone the opportunity to show his erudition, to kill time, to attract attention; but questions have one basic function, the asking for information not already in our possession.

What is a question?

According to Harrah [2002], the following labels for question-types are used by most theorists (examples are taken from the Harrah's paper):

Label	Example
whether	Is two even or odd?
yes-no	Is two a prime number?
which	Which even numbers are prime?
what	What is Church's Thesis?
who	Who is Bourbaki?
why	Why does two divide zero?
deliberative	What shall I do now?
disjunctive	How long is your new proof, or do you have a shorter one?
hypothetical	If you had a proof, how long would it be?
conditional	If you now have a proof, how long is it?
given-that	Given that Turing's Conjecture is provable, is Church's Thesis provable?

The list is by no means exhaustive. One can easily add to it *when*, *where*, *how*..., etc. New types and/or subtypes are distinguished in many theories; the terminology is not well-established.

What is a question? – three levels of analysis

The term ‘question’ can be understood:

syntactically: as referring to a sentence of a particular type, that is, to an interrogative sentence,

semantically: as referring to the semantic content of an interrogative sentence,

pragmatically: as referring to a speech act that is typically performed in uttering interrogative sentences.

Interrogatives

A formal, logical or linguistic, theory of questions should have **interrogative formulas** within its conceptual repertoire. However, the particular shape of these interrogative formulas, or interrogatives for short, is dependent upon how the semantic content of an interrogative sentence/question is conceived.

Some theorists adopt the paraphrase approach. The basic idea is:

The meaning of an interrogative sentence can be adequately characterized by a paraphrase that specifies:

1. the typical use of the sentence, or
2. the relevant illocutionary act performed in uttering the sentence.

Imperative-epistemic paraphrases

Different kinds of paraphrases have been proposed. Among them, imperative-epistemic stances are paired with most thoroughly elaborated theories. For example, the following are paraphrases advocated by Åqvist and Hintikka, respectively:

Let it be the case that I know...
Bring it about that I know...

The dots should be filled by an embedded interrogative sentence. So we have e.g.:

Let it be the case that I know where you bought your car.
Bring it about that I know where you bought your car.

The semantics of the whole paraphrase, however, does not presuppose any semantics of the embedded interrogative. On the contrary, these are semantics of imperative and epistemic expressions that provide a semantic account of the interrogative sentence under consideration.

An example: Hintikka's account

Hintikka interprets questions as requests for information or knowledge; according to his proposal, a question can be paraphrased as an expression which consists of the operator "Bring it about that" followed by the so-called desideratum of the question; the desideratum describes the epistemic state of affairs the questioner wants the respondent to bring about. So the general schema is:

Bring it about that <u>I know [an embedded interrogative]</u>.

where the underlined part constitutes the desideratum. Desiderata of various questions contain such epistemic expressions as "know whether", "know where", "know who", "know when", etc. These concepts of knowledge are explicated by Hintikka in terms of the concept of "knowing that".

Speech acts

When questions are viewed as speech acts, a paraphrase describes the relevant illocutionary act performed. Thus we have:

I hereby ask you...

or

(Please) tell me truly...

and so on. When the paraphrase approach is adopted, it is natural to shape interrogatives by means of expressions of a theory which characterizes the basic elements of the paraphrase.

The independent meaning thesis

Other theorists advocate the independent meaning thesis:

> The meaning / semantic content of an interrogative sentence cannot be adequately characterized in terms of semantics for expressions that belong to other categories.

This is often accompanied with non-reductionism with regard to interrogative formulas: their form is specific, that is, differs from these used to formalize declarative sentences, or (epistemic) imperatives, etc.

The independent meaning thesis – an example

Consider the following:

Which three boys love which two girls?

The question may be interpreted:

1. as asking about three boys and two girls such that each of these girls is loved by exactly one of the boys;
2. as asking about three boys and two girls such that each of the boys loves the two girls;
3. as asking about three boys and some (but at least two and no more than six) girls such that each of the boys loves two of the girls;
4. sometimes the question can also be understood as asking about a complete list of boys and girls fulfilling one of the above conditions.

The effect is not due only to the presence of numerical expressions; consider:

Which boys love which girls?

The independent meaning thesis – an example

Compound numerical interrogative operators disambiguate multiple wh-questions. A compound numerical interrogative operator is not simply a string of interrogative quantifiers: it also includes some constants which, in effect, characterize the relevant scope dependences.

Remark: One may argue that English does not provide interrogatives like:

Which three boys love which two girls? Which at most three kings have ruled which at least two countries?

A reply given by Harrah [2002]: Polish shows that it ought to :)

One more example: Ginzburg's account

Another recently developed approach to questions is based on situational semantics. The old idea, according to which questions are akin to 'open propositions', is conceptualized by Ginzburg by conceiving questions as *propositional abstracts*.

At the start, one considers an universe which contains among its members a class of entities called situations (these are partial, temporally located, actual entities) and a class of entities called states-of-affairs (hereafter: SOA's). In addition, the universe comprises relations as well as possibilities, facts, and outcomes; the latter will not interest us here.

A SOA is a structured object constructed from a relation and an assignment of entities to the argument roles of the relation. (Relations are not conceived as sets of ordered tuples, but as unstructured atomic individuals.) Here is an example of SOA (the notation is self-explanatory):

«Like; {likes: John, is liked: Mary}».

One more example: Ginzburg's account

The role of SOAs is to designate properties that situations might possess. SOA's may be supported by situations; for instance, each situation in which John likes Mary supports the above SOA. There are positive and negative SOA's; the latter are constructed from the former via an adverbial-like operator. *Atomic propositions*, in turn, are defined in terms and situations and SOA's: prop(s,α) is the proposition that s is a situation of the type designated by SOA α. The set of propositions includes atomic propositions and is closed under certain operations of negation, meet and join.

The next step is to define a certain semantic operation of abstraction; the abstracts, then, enrich the universe. There is no space for going into details. Let us only note that the relevant concept of abstraction diverges from these known from Montague semantics or Lambda Calculus; roughly, abstraction is a semantic operation akin to substitution.

One more example: Ginzburg's account

Questions are then conceived as abstracts from propositions. Thus, for example, we have:

1. '$\lambda\{\ \}$prop(s, «Like; likes: John, is liked: Mary»)'
2. '$\lambda\{x\}$prop(s, «Like; likes: x, is liked: Mary»)'
3. '$\lambda\{x, y\}$prop(s, «Like; likes: x, is liked: y»)'

for "Does John like Mary?", "Who likes Mary?" and "Who likes whom?", respectively.

So, what is a question?

1. reductionism
   1. radical: Questions are not linguistic entities; they are identified with: sets of (some kinds of) answers, functions defined on possible worlds, etc.
   2. moderate: Questions are linguistic entities, which can be reduced to expressions of some other categories (or, can be adequately paraphrased as an expression of some other category), for the purposes of obtaining decent logic of questions.
2. non-reductionism: Questions are specific expressions of strictly defined form; they are not reducible to expressions of other categories:

What is an answer?

(*) What is the product of 2 and 2?

1. What a rainy day!
2. Definitely it is not 17.
3. 4.
4. 5.

What is an answer?

It is usually assumed that a question may have many answers and thus the phrase ‘an answer to a question’ does not amount to ‘the true answer to a question.’ In other words, the analyzed answers are usually possible answers; a possible answer may be true or false, or even have no logical value at all.

The standard way of proceeding, then, is to define/characterize some basic category of possible answers. They are called, depending on the theory, direct, or conclusive, or proper, or sufficient, or exhaustive, or complete, etc. Those **principal possible answers**, ppa’s for short, are supposed to satisfy some general conditions, usually expressed in pragmatic terms. For example:

What is an answer?

1. Harrah [1963]: direct answers are replies which are complete and just-sufficient answers.
2. Belnap [1969, p. 124]: direct answers are the answers which "are directly and precisely responsive to the question, giving neither more nor less information than what is called for."
3. Kubiński [1980, p. 12]: direct answers are "these sentences which everybody who understands the question ought to be able to recognize as the simplest, most natural, admissible answers to the question".
4. Hintikka [1978]: a reply is called a conclusive answer if it completely satisfies the epistemic request of the questioner.
5. Ginzburg [1995, p. 461]: ppa's form the "class of responses that a querier would consider optimal."

Nb. A *response* is usually understood as any kind of reaction to a question, verbal as well as non-verbal.

What is an answer?

The ways in which different theories assign ppa's to questions diverge. For example:

1. in the standard solution, ppa's belong to a question: a question is a set of propositions, and these propositions are identified with exhaustive answers to the question.
2. Ginzburg, in turn, proceeds as follows: a class of information items that potentially resolve a question is characterized, and then some conditions are imposed on members of this class in order to explicate what it means that a potentially resolving information item is a resolving answer in a given context.
3. on Hintikka's account, a possible reply is regarded as a conclusive answer if this reply together with the description of the questioner's state of knowledge entails (in the sense of some underlying epistemic logic) the desideratum of the question.
4. Harrah and Kubiński proceed purely syntactically; similarly Belnap (in most cases).

What is an answer?

The following postulates were put forward by Hamblin [1958]:

1. An answer to a question is a statement.
2. Knowing what counts as an answer is equivalent to knowing the question.
3. The possible answers to a question are an exhaustive set of mutually exclusive possibilities.

Hamblin's postulates affected the logic of questions a lot. However, for each postulate there exist theories which violate it – and for a reason.

Terminology: For brevity, instead of "principal possible answers" we shall use the term "direct answers". But please remember that the exact meaning of this term varies from a theory to a theory.

What is an answer? – an example revisited

(*) What is the product of 2 and 2?

1. What a rainy day!
2. Definitely it is not 17.
3. 4.
4. 5.

Plus: answers vs responses.

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How to enrich a formal language with questions?

When one has a formalized language such that questions/interrogatives are not present in the language, there are two ways of adding them to the language:

1. to regard as questions/ interrogatives some (already given) meaningful expressions of a strictly defined form, roughly: to define questions within the language,
2. to extend the language with some question-forming expressions and then to introduce questions/ interrogatives syntactically; the resultant new category of meaningful expressions (i.e. of questions/ interrogatives) is disjoint with the remaining categories.

In what follows we shall employ the second strategy.

Inferential Erotetic Logic, IEL (Wiśniewski [1995])

Basic assumptions:

1. Direct answers are sentences (no free variables allowed).
2. To every question there are at least two direct answers.
3. Every finite and at least two-element set is a set of answers to some question.

The structure of reasoning, simple picture

1. Syntax.
2. Semantics.
3. Logical consequence / derivability.

How to enrich a formal language with questions?

Let J be some formalized language. Let J^* results from J by enriching it with three symbols: ?, {, }.

- Declarative formulas of J^* (d-wffs) are all (and only) formulas of J.
- Erotetic formulas of J^* (or questions of J^*; e-wffs) are all (and only) formulas of the form:

  $$?\{A_1, ..., A_n\}$$

  where:

  $2 \leq n$

  $A_1, ..., A_n$ are syntactically distinct d-wffs

D_{J^*} – the set of all d-wffs of J^*

E_{J^*} – the set of all e-wffs of J^*

dQ – the set of all direct answers to the question Q

Questions of a formalized language vs natural language questions

Wiśniewski [1995]

We claim that questions of a formalized language in which the question-answer relationship is defined syntactically represent questions of a natural language. The relation of representation we have in mind can be characterized as follows: a question Q of a formalized language represents a question Q^* of a natural language construed in such a way that the possible and just-sufficient answers to Q^* have the logical form of direct answers to Q. If a natural-language question has many readings, it has many representations. The richer the underlying formalized language, the more we can represent within it.

Questions of a formalized language vs natural language questions

Any question of the form:

(#) $?\{A_1, A_2, ..., A_n\}$

can be read: Is it the case that A_1, or is it the case that A_2, ..., or is it the case that A_n?

However, sometimes a different reading can be recommended. The scheme (#) is general enough to capture most (if not all) of propositional questions studied in the literature. For example, a simple yes-no question can be formalized as:

$?\{A, \text{non-}A\}$

and read "Is it the case that A?"; the d-wffs A and non-A are the affirmative answer and the negative answer, respectively.

Questions of a formalized language vs natural language questions

- Is it the case that John went to a pub, or John went to a cinema, or John stayed at home?

$$?\{A, B, C\}$$

- Does John love Margaret?

$$?\{A, \text{non-}A\}$$

- Does John love Margaret and Sophie?

$$?\{A \text{ and } B, \text{non-}(A \text{ and } B)\}$$

- Does John love Margaret and does John love Sophie?

$$?\{A \text{ and } B, \text{non-}A \text{ and } B, A \text{ and non-}B, \text{non-}A \text{ and non-}B\}$$

Questions of a formalized language vs natural language questions

- Does John love Margaret?

$$?\{A, \text{non-}A\}$$

simple yes-no question: $?A$

- Does John love Margaret and Sophie?

$$?\{A \text{ and } B, \text{non-}(A \text{ and } B)\}$$

- Does John love Margaret and does John love Sophie?

$$?\{A \text{ and } B, \text{non-}A \text{ and } B, A \text{ and non-}B, \text{non-}A \text{ and non-}B\}$$

binary conjunctive question: $? \pm |A, B|$
conjunctive question: $? \pm |A_1, A_2 \ldots, A_n|$ (Is it the case that A_1, and A_2, and... and A_n?)

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Semantics

It is doubtful if questions should have been assigned Truth or Falsity. But these questions do differ with respect to their semantic properties quite intuitively:

- Is the present king of France bald?
- Is Warsaw the capital of Poland?

Soundness

Question Q is sound if and only iff it has a true direct answer (with respect to the underlying semantics).

Semantics – modest, but still

Suppose that we have the notion of truth defined for the declarative part of J^*. It is obvious that D_{J^*} and E_{J^*} are disjoint.

Partition of D_{J^*}

is an ordered pair $P =< T_P, U_P >$, where T_P and U_P are disjoint and exhaustive (w.r.t. D_{J^*}).

Partition of J^*

is a partition of D_{J^*}.

Observe that a question neither belongs to T_P nor to U_P.

Semantics – modest, but still

Truth (falsity) in a partition

A d-wff A will be called true (false) in a partition $P =< T_P, U_P >$ if A is in T_P (if A is in U_P).

Soundness in a partition

A question Q is sound in a partition $P =< T_P, U_P >$ iff there exists some A in **dQ** such that A is in T_P.

That is, a question is sound in a partition iff it has a true direct answer (in this partition).

Entailment, multiple-conclusion entailment

A declarative formula A is *entailed* by a set of declarative formulas X ($X \models A$) iff for every standard partition $< T, U >$ the following holds: if $X \subseteq T$, then $A \in T$.

A set X of d-wffs is *multiple-conclusion entailed* (mc-entailed) by a set of d-wffs Y ($X || \models Y$) iff for every standard partition $< T, U >$ the following holds: if $X \subseteq T$, then $Y \cap T \neq \emptyset$.

- $X \models A$ iff $X || \models \{A\}$
- $X || \models \{A_1, \ldots, A_n\}$ iff $X \models A_1 \vee \ldots \vee A_n$

$$\{p \vee q\} || \models \{p, q\}$$

$$\{p \vee q\} \; non \models p$$

$$\{p \vee q\} \; non \models q$$

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Erotetic reasoning

In erotetic reasoning an iterrogative conclusion (question-conclusion) follows from premises: either declarative, or interrogative (question-premise) and (possibly) declarative as well. Erotetic reasoning is a good representation of some techniques of problem solving:

1. by reduction of an initial problem to simpler ones,
2. by identifying missing information which is needed in order to solve the initial problem.

Validity of erotetic reasoning is grounded in erotetic counterparts of entailment. In the case of Inferential Erotetic Logic (Wiśniewski [1995]) such counterparts are evocation of questions and erotetic implication.

When it is worth to ask a question?

For example:

1. When it is known, that one of the possibilities holds, but it is not known which one.
2. When asking an auxiliary question is useful in answering an initial question.

When it is worth to ask a question?

On (...) in (...) Ms. Maria Gibson has been shot, the spouse to Mr. J. Neil Gibson, an American politician and billionnaire. Her body has been found in the morning near the bridge over the river, close to the Chateau (...). A revolver of a caliber corresponding to the one with which Ms. Gibson was shot has been found on the floor in the dressing-room of the Gibsons' children governess.

Who killed Maria Gibson?

When it is worth to ask a question?

For example:

1. When it is known, that one of the possibilities holds, but it is not known which one.
   We create a list of suspects and ask which one is the culprit.
2. When asking an auxiliary question is useful in answering an initial question.
   Looking for a culprit we ask: who among the suspects had access to the governess' dressing-room?

In the first case a question arises on the basis of a set of declarative premises, in the second case – on the basis of a previous question (and possibly some declarative premises).

Evocation of questions

1. John loves Margaret and loves Sophie.
   ———
   Does John love Margaret? $\times$
2. John loves Margaret or loves Sophie.
   ———
   Does John love Margaret? $\checkmark$

The criteria:

- If all the declarative premises are true, the question-conclusion must be sound.
- No direct answer to the question-conclusion is entailed by the declarative premises.

Evocation of questions

A set $X = \{A_1, \ldots, A_n, \ldots\}$ of d-wffs *evokes* a question Q (in symbols: $A_1, \ldots A_n, \ldots \vdash_E Q_1$) iff

1. $X || \!\!= \mathbf{d}Q$, and
2. for every $A \in \mathbf{d}Q$: $X \ non || \!\!= \{A\}$

    1. $A \vee \neg A \vdash_E ?A$
    2. $A \vee B \vdash_E ?A$
    3. $A \vee B \vdash_E ?\{A, B\}$
    4. $A \vee B \vdash_E ?\{A \wedge B, A \wedge \neg B, \neg A \wedge B\}$
    5. $A \rightarrow B \vdash_E ?A$
    6. $A \rightarrow B \vdash_E ?B$

$X \vdash_E Q$ – how does it work?

The Sign of Zorro analogue ($P =< T, U >$ is a certain partition of the considered language):

X	Q
$X \subseteq T$	sound in P
$X \not\subseteq T$	sound in P or unsound in P

Q	X
sound in P	$X \subseteq T$ or $X \not\subseteq T$
unsound in P	$X \not\subseteq T$

Erotetic implication

1. Did John go to Paris or to Lyon?
   ___
   Did John go to France? ×
2. Did John go to Paris and did he take his favourite beret?
   ___
   Did John go to Paris? ✓
3. Does John love Margaret or Sophie?
   John does not love Sophie.
   ___
   Does John love Margaret? ✓

The criteria:

- If the initial question is sound and all the declarative premises are true, then the question-conclusion must be sound.
- Each of the direct answers to the question-conclusion narrows down the set of possible true direct answers to the initial question, on the basis of the set of declarative premises.

Erotetic implication

A question Q *erotetically implies* (e-implies) a question Q_1 on the basis of a set $X = \{A_1, \ldots, A_n, \ldots\}$ of d-wffs (in symbols: $Q, A_1, \ldots A_n, \ldots \vdash_{Im} Q_1$) iff

1. for every $A \in \mathbf{d}Q$: $X \cup \{A\} \| \!\!= \mathbf{d}Q_1$, and
2. for every $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$, such that $X \cup \{B\} \| \!\!= Y$

If $X = \emptyset$, then we write: $Q \vdash_{Im} Q_1$.

E-implication: examples

1. $?A \vdash_{Im} ?\neg A$
2. $?(A \# B) \vdash_{Im} ?\pm|A, B|$ (where # is any of the $\vee, \wedge, \rightarrow, \leftrightarrow$)
3. $?\pm|A, B| \vdash_{Im} ?A$
4. $?\pm|A, B| \vdash_{Im} ?B$
5. $?\{\neg A, A \wedge B, A \wedge \neg B\} \vdash_{Im} ?A$
6. $?\{A_1, \ldots, A_n\}, A_1 \vee \ldots \vee A_n \vdash_{Im} ?A_i$ (where $1 \leq i \leq n$)
7. $?\{A_1, \ldots, A_n\}, A_1 \leftrightarrow B_1 \wedge \ldots \wedge A_n \leftrightarrow B_1 \vdash_{Im} ?\{B_1, \ldots, B_n\}$
8. $?\{A_1, \ldots, A_n\}, B \rightarrow A_1 \vee \ldots \vee A_i, C \rightarrow A_{i+1} \vee \ldots \vee A_n, B \vee C$
$\vdash_{Im} ?\{B, C\}$

$Q, X \vdash_{Im} Q_1$ – how does it work?

The Sign of Zorro analogue, but not so exact ($P =< T, U >$ is a certain standard partition of the considered language):

Q	X	Q_1
sound in P	$X \subseteq T$	sound in P
unsound in P	$X \subseteq T$	unsound in P
sound in P	$X \not\subseteq T$	sound in P or unsound in P
unsound in P	$X \not\subseteq T$	sound in P or unsound in P

Q_1	X	Q
sound in P	$X \subseteq T$	sound in P
unsound in P	$X \subseteq T$	unsound in P
sound in P	$X \not\subseteq T$	sound in P or unsound in P
unsound in P	$X \not\subseteq T$	sound in P or unsound in P

E-implication: non-transitivity

$?(A \wedge B) \vdash_{Im} ? \pm |A, B|$
$? \pm |A, B| \vdash_{Im} ?A$

but:

$?(A \wedge B) \nvdash_{Im} ?A$

Eliminating vs narrowing down (Wiśniewski [2013])

Elimination

A set X of wffs eliminates a set Y of wffs iff: if all the elements of X are true, then all elements of Y are false (w.r.t. underlying semantics).

Narrowing down

A set X of wffs narrows down a set Y of wffs iff there exists a non-empty proper subset Y^* of Y such that X entails the disjunction of all the elements of Y^*.

$X = \{p, p \rightarrow q \vee r\}$
$Y = \{q, r, t\}$
$Z = \{\neg q\}$

Falsificationist e-implication (Grobler [2006])

A question Q *fe-implies* a question Q_1 on the basis of a set $X = \{A_1, \ldots, A_n, \ldots\}$ of d-wffs (in symbols: $Q, A_1, \ldots A_n, \ldots \vdash_{Imf} Q_1$) iff

1. for every $A \in \mathbf{d}Q$: $X \cup \{A\} || \!\models \mathbf{d}Q_1$, and
2. there exists $B \in \mathbf{d}Q_1$, że such that for some non-empty proper subset $Y = \{C_1, \ldots, C_n\}$ of $\mathbf{d}Q$ the following holds: $X \cup \{B\} \models \{\neg C_1, \ldots, \neg C_n\}$

Weak e-implication (Urbański et al. [2016])

A question Q *w-implies* a question Q_1 on the basis of a set $X = \{A_1, \ldots, A_n, \ldots\}$ of d-wffs (in symbols: $Q, A_1, \ldots A_n, \ldots \vdash_{Imf} Q_1$) iff

1. for every $A \in \mathbf{d}Q$: $X \cup \{A\} || \!\!= \mathbf{d}Q_1$, and
2. for some $B \in \mathbf{d}Q_1$ there exists a non-empty proper subset Y of $\mathbf{d}Q$, such that $X \cup \{B\} || \!\!= Y$

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Internal Question Processing

Questions are not only asked and then (possibly) answered, but they are also internally processed.

Internal question processing (hereafter: IQP) is a cognitive phenomenon which is distinct from question answering. When a question is internally processed, the immediate outcome need not be an answer to this question: an 'inference' performed on a question can lead to another question, which may be 'send' by an agent either to itself or to a certain external source of information and then answered, but can also be processed further in an analogous way. The relevant transformations of questions usually facilitate question answering and problem-solving. Sometimes, however, they finally result in a plausible answer/solution to a question/problem.

Ultimate and Distributed Internal Questions Processing

As long as ultimate IQP is concerned, questions are transformed into consecutive questions until a question which can be answered in only one (rational) way is arrived at. No requests for additional information are sent.

In the case of distributed IQP requests for additional information are sent, and questions are transformed into further questions depending on how previous information requests have been fulfilled.

Distributed IQP: An example

– Where did Andrew leave for: Paris, London or Moscow?
– If these are the options, then he departed in the morning or in the evening. And if in the morning, he left for Paris or London; otherwise he left for Moscow.
– [So] When did he depart: in the morning, or in the evening?
– In the morning.
– [So] Did Andrew leave for Paris, or for London?
– Andrew takes his famous umbrella only when traveling to London.
– [So] Did he take it?
– Yes.
– [So] Andrew left for London.

Distributed Internal Questions Processing

In the case of distributed IQP requests for additional information are sent, and questions are transformed into further questions depending on how previous information requests have been fulfilled.

The following principle seems to govern effective IQP:

Decomposition Principle

Transform a principal question into auxiliary questions in such a way that plausible answers to the auxiliary questions yield a plausible answer to the principal question, and consecutive auxiliary questions depend on answers to previous auxiliary questions.

As long as distributed IQP is concerned, the main difficulty stems from the fact that an auxiliary question has many possible (direct) answers and one does not know in advance which of them is plausible; this is resolved only after the relevant request for information is satisfied.

The example, 1

Q *Where did Andrew leave for: Paris, London, or Moscow?*

D1 If these are the options, then he departed in the morning or in the evening.

D2 And if in the morning, he left for Paris or London; otherwise he left for Moscow.

D3 Andrew takes his famous umbrella only when traveling to London.

The example, 1

Q *Where did Andrew leave for: Paris, London, or Moscow?*

D1 If these are the options, then he departed in the morning or in the evening.

D2 And if in the morning, he left for Paris or London; otherwise he left for Moscow.

D3 Andrew takes his famous umbrella only when traveling to London.

Q1 *Did Andrew depart in the morning, or in the evening?*

The example, 1

Q *Where did Andrew leave for: Paris, London, or Moscow?*

D1 If these are the options, then he departed in the morning or in the evening.

D2 And if in the morning, he left for Paris or London; otherwise he left for Moscow.

D3 Andrew takes his famous umbrella only when traveling to London.

Q1 *Did Andrew depart in the morning, or in the evening?*

A1 **In the morning.**

Q2 *Did Andrew leave for Paris, or for London?*

The example, 1

Q *Where did Andrew leave for: Paris, London, or Moscow?*

D1 If these are the options, then he departed in the morning or in the evening.

D2 And if in the morning, he left for Paris or London; otherwise he left for Moscow.

D3 Andrew takes his famous umbrella only when traveling to London.

Q1 *Did Andrew depart in the morning, or in the evening?*

A1 **In the morning.**

Q2 *Did Andrew leave for Paris, or for London?*

Q3 *Did Andrew take his famous umbrella?*

The example, 1

- Q *Where did Andrew leave for: Paris, London, or Moscow?*
- D1 If these are the options, then he departed in the morning or in the evening.
- D2 And if in the morning, he left for Paris or London; otherwise he left for Moscow.
- D3 Andrew takes his famous umbrella only when traveling to London.
- Q1 *Did Andrew depart in the morning, or in the evening?*

- A1 **In the morning.**
- Q2 *Did Andrew leave for Paris, or for London?*
- Q3 *Did Andrew take his famous umbrella?*

- A2 **No, he didn't.**
- D4 Andrew left for Paris.

The example, 2

Q *Where did Andrew leave for: Paris, London, or Moscow?*

D1 If these are the options, then he departed in the morning or in the evening.

D2 And if in the morning, he left for Paris or London; otherwise he left for Moscow.

D3 Andrew takes his famous umbrella only when traveling to London.

Q1 *Did Andrew depart in the morning, or in the evening?*

A1 **In the morning.**

Q2 *Did Andrew leave for Paris, or for London?*

Q3 *Did Andrew take his famous umbrella?*

A2 **Yes, he did.**

D4 Andrew left for London.

The example, 3

Q *Where did Andrew leave for: Paris, London, or Moscow?*

D1 If these are the options, then he departed in the morning or in the evening.

D2 And if in the morning, he left for Paris or London; otherwise he left for Moscow.

D3 Andrew takes his famous umbrella only when traveling to London.

Q1 *Did Andrew depart in the morning, or in the evening?*

A1 **In the evening.**

D4 Andrew left for Moscow.

The example, scenario

Where did Andrew leave for: Paris, London, or Moscow?
If these are the options, then he departed in the morning or in the evening.
And if in the morning, he left for Paris or London; otherwise he left for Moscow.
Andrew takes his famous umbrella only when traveling to London.
Did Andrew depart in the morning, or in the evening?

In the morning.
Did Andrew leave for Paris, or for London?
Did Andrew take his famous umbrella?

No
Andrew left for Paris

Yes
Andrew left for London

In the evening.
Andrew left for Moscow.

$?\{p, q, r\}$

$s \vee t$

$s \rightarrow p \vee q$

$t \rightarrow r$

$u \rightarrow q$

$?\{s, t\}$

s	t
$?\{p, q\}$	r
$?\{\neg u, u\}$	

$\neg u$	u
p	q

Erotetic search scenarios: intuitions

- An erotetic search scenario (e-scenario) for Q has a tree-like structure with Q as the root and possible (direct) answers to Q as leaves.
- Auxiliary questions enter an e-scenario on the condition they are erotetically implied.
- An auxiliary question either has another question as the immediate successor (serves as an 'erotetic premise' only) or has all its direct answers as the immediate successors (performs the role of a query).
- In the latter case the immediate successors represent the possible ways in which the relevant request for information can be satisfied, and the structure of an e-scenario shows what further information requests (if any) are to be satisfied in order to arrive at an answer to Q.

Viewed pragmatically, an e-scenario provides us with conditional instructions which tell us what questions should be asked and when. Moreover, an e-scenario shows where to go if such-and-such direct answer to a query appears to be acceptable and does it with respect to any direct answer to each query.

E-scenarios: Golden Path Theorem

Wiśniewski [2003]

If an initial question Q of a given e-scenario Φ has a true direct answer and if all the declarative premises of Φ are true, then at least one path of Φ leads to a true answer to Q; what is more, all d-wffs of this path are true and all e-formulas of this path have true direct answers as well.

E-scenarios

- An e-scenario gives information when and what question should be asked (relative to the initial question and initial premises).
- What is more, it ensures that all subsequent questions asked would be relevant to the initial question.
- E-scenarios also guarantee that each subsequent question asked is a step towards the answer to the initial question.
- The Golden Path Theorem guarantees that for a strategy expressed by an e-scenario there exist at least one such path that ends with the answer to the initial question which is true (relative to the initial premises).
- A strategy presented by an e-scenario is flexible in the sense that it can be modified and rearranged by the embedding procedure.

Difficult deductions: Erotetic Reasoning Test, task 1 (The Bomb)

Introduction: In the capital of a certain country someone placed bomb in the palace of the king. The best royal engineer, who arrived immediately, established the following evidence:

1. There are three wires in the bomb: green, red and orange.
2. To disarm the bomb either the green or the red wire must be cut. Cutting of the wrong wire will cause an explosion.
3. If the bomb has been placed by Steve, then it will be disarmed by cutting the green wire.
4. If the bomb has been placed by John, then it will be disarmed by cutting the red wire. Moreover, no one but John would use the red wire.
5. If the bomb has been placed on an even day of the month, then the culprit is Steve.
6. The bomb has been placed either by Steve, or by John, or by someone else.

ER 1

Instruction: To each of the following questions you may get two answers: 'yes' or 'no'. Mark only one question – that one, to which each answer (no matter 'yes' or 'no') allows to decide which wire should be cut in order to disarm the bomb:

- ☐ Is the bomb placed on an even day of the month?
- ☐ Is the bomb placed by Steve?
- ☐ Is the bomb placed by John?
- ☐ Is the bomb placed by someone else than Steve or John?

ER 1

Which wire to cut?

1. There are three wires in the bomb: green, red and orange.
2. To disarm the bomb either the green or the red wire must be cut. Cutting of the wrong wire will cause an explosion.
3. If the bomb has been placed by Steve, then it will be disarmed by cutting the green wire.
4. If the bomb has been placed by John, then it will be disarmed by cutting the red wire. Moreover, no one but John would use the red wire.
5. If the bomb has not been placed on an even day of the month, then the culprit is Steve.
6. The bomb has been placed either by Steve, or by John, or by someone else.

Even day? Steve? John? Someone else?

ER 1

Is the bomb placed by John?

- **Yes** $\Rightarrow$ Cut the **red wire**.
- **No** $\Rightarrow$ No matter who placed the bomb, it cannot be disarmed by cutting the red wire. So, cut the **green wire**.

Is the bomb placed by Steve?

- **Yes** $\Rightarrow$ Cut the **green wire**.
- **No** $\Rightarrow$ The solution still depends on who placed the bomb – John or someone else.

Subject 100, solution: Czy bombę podłożył Ignacy?

Rozważyłam wszystkie możliwe odpowiedzi. Jeśli to Ignacy, należy przeciąć czerwony. Jeżeli nie, to należy przeciąć zielony, ponieważ tylko te dwa kolory rozbrajają bombę, a tylko Ignacy użyłby czerwonego, zatem ktoś inny użyłby zielonego.

Jest to jedyne pytanie, na które odpowiedź daje jednoznaczne rozwiązanie. przy użyciu którejś z pozostałych wskazówek istnieje ryzyko, że będzie trzeba odpowiedzieć na jeszcze inne pytania.

ER 1

Subject 81, solution: Czy bombę podłożono w dzień parzysty?

Ta odpowiedź, ponieważ jeśli bomby nie podłożono w dzień parzysty, to zrobił to Stefan. I będzie wiadomo kto to zrobił, trzeba tylko zwrócić uwagę czy był to dzień parzysty czy nieparzysty.

Subject 78, solution: Czy bombę podłożono w dzień parzysty?

Bo jeżeli to był dzień parzysty to znany jest sprawca.

Subject 101, solution: Czy bombę podłożono w dzień parzysty?

Obojętnie które pytanie zadamy wykluczy ono jednego podejrzanego i konieczne będzie zadanie drugiego pytania, które rozstrzygnie kto podłożył bombę

Subject 82, solution: Czy bombę podłożono w dzień parzysty?

Nie wiemy czy bomba w ogóle wybuchła i nie wiemy kto podłożył bombę.

ER 1

Subject 91, solution: Czy bombę podłożył Stefan?

Ponieważ jeśli bombę podłożył Stefan, to unieszkodliwia ją przecięcie zielonego kabelka.

Subject 103, solution: Czy bombę podłożył Stefan?

Jeśli uzyskamy na to pytanie odpowiedź twierdzącą, to będziemy wiedzieli, że trzeba przeciąć zielony kabelek. Jeśli przeczącą to zostanie nam tylko jedna możliwość – czerwony kabelek i dodatkowo będziemy wiedzieć, że winny jest Ignacy.

ER 1 – implications

Initial question: $Q = ?\{p, q, v\}$

1. $p \vee q$
2. $s \rightarrow p$
3. $r \leftrightarrow q$
4. $\neg t \rightarrow s$
5. $r \perp s \perp u$

1. $?\{t, \neg t\}$
2. $?\{r, \neg r\}$
3. $?\{s, \neg s\}$
4. $?\{u, \neg u\}$

	Q, X, Q_1	e-implication
8.	$Q, \neg t \rightarrow s, s \rightarrow p, ?\{t, \neg t\}$	$*\mathbf{Im}$, $\mathbf{Im_w}$, $*\mathbf{Im_f}$
9.	$Q, p \vee q, r \leftrightarrow q, ?\{r, \neg r\}$	$\mathbf{Im}$, $\mathbf{Im_w}$, $\mathbf{Im_f}$
10.	$Q, s \rightarrow p, ?\{s, \neg s\}$	$*\mathbf{Im}$, $\mathbf{Im_w}$, $*\mathbf{Im_f}$
11.	$Q, p \vee q, r \perp s \perp u, r \leftrightarrow q, ?\{u, \neg u\}$	$*\mathbf{Im}$, $\mathbf{Im_w}$, $\mathbf{Im_f}$

ER 1, scenario 1 – canonical e. i.

$$?\{p, q, v\}$$
$$p \vee q$$
$$s \rightarrow p$$
$$r \leftrightarrow q$$
$$\neg t \rightarrow s$$
$$r \bot s \bot u$$
$$?\{r, \neg r\}$$

r	$\neg r$
q	$\neg q$
	p

ER 1, scenario 2 – weak e. i.

$?\{p, q, v\}$

$p \vee q$

$s \rightarrow p$

$r \leftrightarrow q$

$\neg t \rightarrow s$

$r \perp s \perp u$

$?\{s, \neg s\}$

s	$\neg s$
p	???

ER 1, scenario 2 – weak e. i., continued

$?\{p, q, v\}$

$p \vee q$

$s \rightarrow p$

$r \leftrightarrow q$

$\neg t \rightarrow s$

$r \bot s \bot u$

$?\{s, \neg s\}$

s	$\neg s$
p	$?\{u, \neg u\}$

u	$\neg u$
$\neg q$	r
p	q

ER 1, scenario 3 – canonical e. i. again

$?\{p, q, v\}$

$p \vee q$

$s \rightarrow p$

$r \leftrightarrow q$

$\neg t \rightarrow s$

$r \perp s \perp u$

$?\{s \wedge u, s \wedge \neg u, \neg s \wedge u, \neg s \wedge \neg u\}$

$?\{s, \neg s\}$

s	$\neg s$
p	$?\{u, \neg u\}$

u	$\neg u$
$\neg q$	r
p	q

ER 1, scenario 4 – canonical e. i. once more

$$?\{p, q, v\}$$
$$p \vee q$$
$$s \rightarrow p$$
$$r \leftrightarrow q$$
$$\neg t \rightarrow s$$
$$r \bot s \bot u$$
$$?\{s, \neg s \wedge u, \neg s \wedge \neg u\}$$

s	$\neg s \wedge u$	$\neg s \wedge \neg u$
p	u	$\neg u$
	$\neg q$	r
	p	q

Difficult deductions: Erotetic Reasoning Test, task 2 (The Tarts)

1. Only three characters were seen near the tray: the March Hare, the Hatter, and the Dormouse.
2. If the March Hare or the Dormouse eat at the Queen's parties, they leave lots of crumbs near the rose bushes.
3. If the March Hare tells jokes during parties, he never eats.
4. The King of Hearts stays till the end of only these parties, at which the March Hare doesn't tell jokes (although even then the King sometimes leaves earlier).
5. The March Hare and the Dormouse never eat from silver trays.
6. The Hatter eats exclusively from silver trays.
7. The Dormouse slept through the whole party.

1. Crumbs? 2. March Hare the joker?
3. King of Hearts till the end? 4. Silver tray?

ER 2

Subject 102, solution: Were the tarts placed on a silver tray?

Dormouse slept trough the whole party, so he couldn't eat the tarts. He and March Hare do not eat from silver trays. So, if the answer is NO, March Hare eat them. If the answer is YES, then the Hatter ate them, because only he eats from silver trays.

Subject 88, solution: Were the tarts placed on a silver tray?

If we establish that the tarts were placed on the silver tray, only the Hatter could eat them.

Subject 86, solution: Were the tarts placed on a silver tray?

If the tarts were placed on the silver tray, then March Hatter and Dormouse don't eat them, only the Hatter.

ER 2

Subject 94, solution: Are there any crumbs near the rose bushes?

If there are crumbs, the tarts were eaten by March Hare (because Dormouse slept). If there are no crumbs, it was the Hatter.

Subject 85, solution: Did the King of Hearts stay till the end of the party?

The King of Hearts will know the answer, if he stayed till the end. If no, then the Hatter ate the tarts.

Subject 79, solution: Did the March Hare tell jokes at the party?

If March Hare tells jokes, then there are no tarts at the party, so he couldn't eat them. The King also couldn't, because he attends only those parties at which March Hare does not tell jokes. Then only Dormouse and the Hatter left, and Dormouse couldn't eat them, because he slept.

ER 2

$?\{p, q, w\}$
$p \vee q \vee w$
$p \vee q \rightarrow r$
$s \rightarrow \neg p$
$\neg s \rightarrow k$
$u \rightarrow \neg(p \vee q)$
$w \rightarrow u$
$\neg q$
$?\{u, \neg u\}$

u	$\neg u$
$\neg(p \vee q)$	$\neg w$
w	p

MH, H or D?
Either MH or H, or D.
If MH or D, then crumbs.
If MH jokes, never eats.
If no MH jokes, then KoH stays.
If silver, then no MH or D.
If H, then silver.
D slept.
Silver?

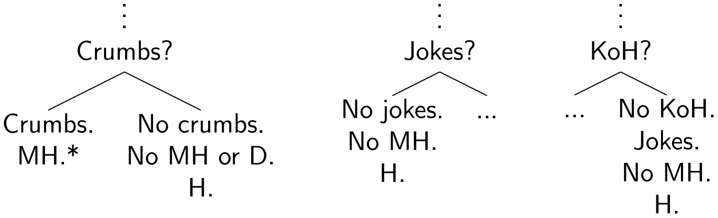
Silver.	No silver.
No MH or D.	No H.
H.	MH.

(subject 102)

MH, H or D?
Either MH or H, or D.
If MH or D, then crumbs.
If MH jokes, never eats.
If no MH jokes, then KoH stays.
If silver, then no MH or D.
If H, then silver.
D slept.
Silver?

Silver	No silver.
No MH or D.	No H.
H.	MH.

(subjects 86, 88)



(subject 94) (subject 79) (subject 85)

ER 2

Initial question: $Q = ?\{p, q, w\}$

1. $p \vee q \vee w$
2. $p \vee q \rightarrow r$
3. $s \rightarrow \neg p$
4. $\neg s \rightarrow k$
5. $u \rightarrow \neg(p \vee q)$
6. $w \rightarrow u$
7. $\neg q$

1. $?\{r, \neg r\}$
2. $?\{s, \neg s\}$
3. $?\{k, \neg k\}$
4. $?\{u, \neg u\}$

	Q, X, Q_1	implications
1.	$Q, p \vee q \rightarrow r, p \vee q \vee w,$ $?\{r, \neg r\}$	$\mathbf{Im}^*$, $\mathbf{Im_w}$, $\mathbf{Im_f}$
2.	$Q, s \rightarrow \neg p, \neg q, p \vee q \vee w,$ $?\{s, \neg s\}$	$\mathbf{Im}^*$, $\mathbf{Im_w}$, $\mathbf{Im_f}$
3.	$Q, s \rightarrow \neg p, \neg s \rightarrow k, \neg q,$ $p \vee q \vee w, ?\{k, \neg k\}$	$\mathbf{Im}^*$, $\mathbf{Im_w}$, $\mathbf{Im_f}$
4.	$Q, u \rightarrow \neg(p \vee q), w \rightarrow u,$ $p \vee q \vee w, \neg q, ?\{u, \neg u\}$	$\mathbf{Im}$, $\mathbf{Im_w}$, $\mathbf{Im_f}$

$\mathbf{Im}$ vs $\mathbf{Im_f}$ vs $\mathbf{Im_w}$

A question Q is disjoint iff all the direct answers to Q are mutually exclusive.

Theorem 1.

If $\mathbf{Im}(Q, X, Q_1)$, then $\mathbf{Im_w}(Q, X, Q_1)$

Theorem 2.

If $\mathbf{Im}(Q, X, Q_1)$, then $\mathbf{Im_f}(Q, X, Q_1)$, provided that Q is a disjoint question.

Theorem 3.

If Q is a disjoint question, then $\mathbf{Im_w}(Q, X, Q_1)$ iff $\mathbf{Im_f}(Q, X, Q_1)$.

Vampires, zombies and ATMs (Urbański, Łupkowski [2010])

On a certain island the inhabitants have been bewitched by some kind of magic. Half of them turned into zombies, the other half turned into vampires. The zombies and the vampires of this island do not behave like the conventional ones (if any): they move about and talk in as lively a fashion as do the humans, and the vampires even prefer drinking strong mocca over anything else. It's just that the zombies of this island always lie and the vampires of this island always tell the truth. What is also important, both vampires and zombies never miss a reasonable opportunity to tell the truth or to lie, respectively. Thus they always do their best to answer questions addressed to them.

A native named Eugene has been suspected of an attempt to break in an ATM near the police station. The case has been assigned to Inspector Negombo (a vampire) of local police force. His first task was to establish if the accused is a vampire or a zombie. Inspector Negombo was clever enough to determine that Eugene is a vampire on the basis of the suspect's answer to a single question. What was Negombo's question?

Vampires, zombies and ATMs – data

1. Every native is either a vampire or a zombie.
2. Every native who is a vampire utters true sentences.
3. Every native who utters a true sentence is a vampire.
4. Eugene is a native of the island.

More handy version:

2′. For every native x, if x gives back a true answer to a posed question, then x is a vampire.

3′. For every native x, if x is a vampire, then x gives back a true answer to a posed question.

Vampires, zombies and ATMs, formalized

Let $V(x)$, $Z(x)$, $N(x)$ stand for expressions: "x is a vampire", "x is a zombie", "x is a native of the island" respectively, let $U(x, A_i, ?\{A_1, \ldots, A_n\})$ stand for "x gives back an answer A_i to the question $?\{A_1, \ldots, A_n\}$", and let the constant a represent Eugene. T stands for the lack of factual knowledge. What R stands for?

$$?\{V(a), Z(a), \mathsf{T}\}$$
$$\forall x(N(x) \rightarrow V(x) \perp Z(x))$$
$$\forall x(U(x, R, ?R) \rightarrow V(x))$$
$$\forall x(V(x) \rightarrow U(x, R, ?R))$$
$$N(a)$$
$$?V(a)$$
$$?U(a, R, ?R)$$

Left branch:

$$U(a, R, ?R)$$
$$V(a)$$

Right branch:

$$\neg U(a, R, ?R)$$
$$\neg V(a)$$
$$Z(a)$$

R stands for the formula, for example, $V(a) \vee \neg V(a)$.

The story continues

It would be quite simple now to solve the problem: just ask Eugene if he really did try to break in the ATM. But this simple plan has been ruined by discovery that one premise on which Negombo's inferences were dependent is false: Eugene was not a native of the island. He came there as an immigrant from the nearby island, inhabited exclusively by humans, who are totally unpredictable as for the truth or falsity of what they tell. Moreover, Eugene refused to give any sort of testimony.

Further evidence

5. If Eugene did run short of money, then he attempted to break in an ATM or borrowed some money from Eustace.
   $Sh(a) \rightarrow At(a) \vee Br(a, c)$
6. If Eugene didn't run short of money, then he went shopping or visited his favourite pub.
   $\neg Sh(a) \rightarrow Sp(a) \vee Pb(a)$
7. If Eugene attempted to break in an ATM or went shopping, then he has been seen in a local mall.
   $At(a) \vee Sp(a) \rightarrow Ml(a)$
8. If Eugene borrowed some money from Eustace or visited his favourite bar, then he hasn't been seen in a local mall.
   $Br(a, c) \vee Pb(a) \rightarrow \neg Ml(a)$

Second scenario

$?\{At(a), \neg At(a), \mathsf{T}\}$
$Sh(a) \to At(a) \vee Br(a,c)$
$\neg Sh(a) \to Sp(a) \vee Pb(a)$
$At(a) \vee Sp(a) \to Ml(a)$
$Br(a,c) \vee Pb(a) \to \neg Ml(a)$
$?Ml(a)$

- $Ml(a)$
  $\neg(Br(a,c) \vee Pb(a))$
  $\neg Br(a,c)$
  $\neg Pb(a)$
  $?Sh(a)$
  - $Sh(a)$
    $At(a) \vee Br(a,c)$
    $At(a)$
  - $\neg Sh(a)$
    $Sp(a) \vee Pb(a)$
    $Sp(a)$
    T
- $\neg Ml(a)$
  $\neg(At(a) \vee Sp(a))$
  $\neg At(a)$

The story continues

“If he’s guilty and if this is my lucky day, I’ll send him to the court in two questions”, Negombo said to himself. “If he’s been in the mall but didn’t run short of money then my information is insufficient and I will need new evidence. If he hasn’t been in the mall, he’s innocent. Well, we’ll see”.

He ordered one of his lieutnants to conduct an interrogation according to this plan (nb: both queries of this scenario, that is $?Ml(a)$ and $?Sh(a)$, might demand plans for investigation in the form of e-scenarios on themselves). The lieutnant soon reported the outcome: Elyssa answered first query with “No”. Eugene hasn’t been in the mall. He’s innocent!

Third scenario – hidden agenda

$L(x, A)$: "x lies saying A"; $S(x, B, A)$: "x expresses the set of behaviours B while saying A"; b: Elyssa.

$$?L(b, \neg MI(a))$$
$$U(b, \neg MI(a), ?MI(a))$$
$$\forall x(S(x, B_1, A) \rightarrow L(x, A))$$
$$\ldots$$
$$\forall x(S(x, B_n, A) \rightarrow L(x, A))$$
$$\forall x(\neg S(x, B_1, A) \wedge \ldots \wedge \neg S(x, B_n, A) \rightarrow \neg L(x, A))$$
$$?\{L(b, \neg MI(a)), \neg L(b, \neg MI(a)), S(b, B_1, \neg MI(a))\}$$
$$?S(b, B_1, \neg MI(a))$$

$$S(b, B_1, \neg MI(a)) \qquad \neg S(b, B_1, \neg MI(a))$$
$$L(b, \neg MI(a)) \qquad \ldots$$
$$?S(b, B_n, \neg MI(a))$$

$$S(b, B_n, \neg MI(a)) \qquad \neg S(b, B_n, \neg MI(a))$$
$$L(b, \neg MI(a)) \qquad \neg L(b, \neg MI(a))$$

The story ends

This scenario shows that the only question that Negombo should actually pose to Elyssa is "Has Eugene been in the mall?" although it is known what will the answer be. All the remaining questions play the role of milestones on Negombo's way of thinking in solving the initial problem. Notice that they are concerned not with the content of Elyssa statement but with the way she provided that statement.

Elyssa repeated her previous testimony that Eugene has not been in the mall. But saying this she expressed a set of behaviours characteristic for a liar (say that they were microexpressions of her lips indicating disbelief in what she has been saying; an interested reader may have a look at the example of such microexpression on Paul Ekman's page: `http://www.paulekman.com`). On this basis Negombo determined that she is lying that Eugene has not been in the mall. To finish his investigation quickly he decided to employ ethically disputable means. He produced a fake witness (who testified that he has seen Eugene in the mall) and confronted Elyssa with him. Elyssa finally admitted that she was lying and that Eugene in fact has been in the mall on that particular day. Her statement has been recorded. The case has been sent to the court.

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